

Deficiencies in Mathematics in Primary Schools

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Introduction:

The following is gleaned from many years of working in government primary schools in Tanzania. It also comes from experience of child development and how children learn.

How children learn

Children learn by using and seeing 'concrete' items and then slowly moving towards the 'abstract'. So, children learn by touching and seeing real things (bottle tops for counting, sticks in bundles of ten for 'place value' of *makumi*), and slowly move away from using 'concrete' to being able to work with 'abstract' alone (knowing that '4' is the abstract representation of four bottle tops etc, and that '12' means one bundle of ten, and two units/single item)

A thorough understanding of the above is fundamental to any teaching of mathematics to children. Starting with 'concrete' is very easy to incorporate into lessons, and the use of 'concrete' makes learning enjoyable and fun. This is the case not just for Standard I and II pupils, but holds true for pupils in the higher standards as well, although in those higher standards the 'concrete' items will most probably combine with the abstract much more quickly than with younger pupils.

Skills and knowledge

All primary education is a delicate mixture of developing skills within pupils and giving those pupils knowledge. The decision of what skills with which to equip children is something for a government to decide. However, in primary education the skills which are developed consciously in the teaching of mathematics are: counting skills; skills in working with numbers - adding, subtracting, dividing and multiplying; place value skills; thinking and problem solving skills; application skills (applying knowledge gained); skills in measuring; skills in sorting and classifying; skills of estimating; visual/spacial awareness skills; skills in logical progression; skills of logical deduction.

Relevance to pupils' lives

Those skills very much relate to mathematics being part of everyday life. For instance, whilst shopping we are adding and subtracting with payment and change; we might be working out percentages with discounts on offer; we might be using multiplication if we buy several of one item etc. The most relevant and interesting way to teach mathematics is to use real-life situations. This will use 'narrative' questions – so that pupils are given a scenario and then have to work something out.

Data Collected, and evidence of Standard IV deficiencies.

The data comes from the analysis of the 2012 National Examination for Standard IV which covered four schools and 186 pupils.

The data comprises questions that have been categorised by topic (that is topics in the syllabus). The marks of each pupil are recorded. The resultant chart, therefore, shows the overall performance for each topic, and shows clearly topics where the majority of the pupils were unable to answer the question adequately. The chart of data for Standard IV is in Appendix 1.

In summary the topics which pupils fail to understand sufficiently are:

- Processes: addition, subtraction, multiplication and division
- Place value: units, tens, hundreds
- Problem solving: word/narrative problems
- Measurement: length, area, perimeter, time, mass
- Fractions
- Graphing

Data Collected, and evidence of Standard VII deficiencies.

The data comes from the analysis of five Standard VII national and mock examination papers in Mathematics from 13 schools. A total of 707 student papers were analysed from those examinations. Data was also collected from working in Mathematics classrooms with Standard 7 teachers.

The data comprises questions that have been categorised by topic (that is topics in the syllabus). The marks of each pupil are recorded. The resultant chart, therefore, shows the overall performance for each topic, and shows clearly topics where the majority of the pupils were unable to answer the question adequately. The chart of data for Standard VII is in Appendix 2.

In summary the topics which pupils fail to understand sufficiently are:

- Processes: addition, subtraction, multiplication and division- particularly with decimals and negative numbers
- Extended number facts,
e.g. times tables, $4 \times 5 = 20$, $40 \times 5 = 200$. Multiplying and dividing by 10, 100 & 1,000
- Place value: particularly with decimals
- Problem solving: word/narrative problems
- Measurement: length, area, perimeter, time, mass
- Graphing
- Algebra
- Perimeter of a circle, irregular shapes

Reasons for the deficiencies in Standard IV and Standard VII pupils' understanding and learning – vis-à-vis the content of the syllabus

Standard IV:

- Processes:
 - Lack of use of 'concrete' materials when teaching numbers and number facts
 - Lack of knowledge of basic number concepts e.g. times tables, number bonds to 10 and 20
 - Use of abstract mathematics too quickly
 - No use of 0 as a place holder in calculations
- Place Value:
 - No use of 0 as a place holder
 - Setting out mathematical equations with numbers in the wrong column
- Problem solving:
 - Lack of understanding of the problem solving process
 - Lack of understanding of mathematical processes, e.g.- addition, subtraction, multiplication and division
- Measurement:
 - Lack of understanding of the concepts- What is perimeter? What is area? Jumping into formulaic methods, e.g. $A = L \times W$ before understanding the reality of, e.g. length and width.
- Fractions:
 - Lack of knowledge what fractions are
 - Lack of knowledge of how to read fractions
 - Fractions of numbers
- Graphing
 - Lack of knowledge of what a graph is.
 - No understanding of how to read a graph

Standard VII:

- Processes:
 - Lack of basic number knowledge e.g. times tables
 - Lack of use of 'concrete' materials when teaching number facts in early years
 - Use of abstract mathematics too quickly
 - Lack of 'place value' knowledge
 - No use of 0 as a place holder
 - Setting out mathematical equations with numbers in the wrong column
- Extended number facts:
 - Lack of understanding of basic number knowledge
 - Rote learning of mathematical information without 'concrete' examples and relevance to real life
- Problem solving:
 - Lack of understanding of the problem solving process:
 - Identifying what the question is asking, the mathematics involved in the question, knowing what strategies to use to solve a problem
 - Lack of understanding of practical application of mathematical processes and knowledge, e.g.- addition, subtraction, multiplication, division, fractions, measurement.
- Measurement:
 - Lack of understanding of the concepts. Jumping into formulaic methods (Circumference = $2 \pi r$) before understanding what the reality is, such as what a circumference is.
 - Lack of knowledge of g < kg, ml < litre, cm < m, m < km, minutes < hour, seconds < 1 minute
- Graphing
 - Lack of knowledge of what a graph is. Graphs show data in a clear, visual way. Graphs are used to extract information. In the syllabus graphs are used as a basis for calculations which is a distortion of their function.
 - Lack of interpretive skills to work with graphs or to draw graphs
 - No understanding of how to read a graph
- Algebra
 - Lack of understanding what algebra is and what it is for.
 - Lack of the basic mathematical knowledge and skills needed to understand and work with algebra
- Perimeter
 - As seen in the Standard IV examinations pupils do not understand the concept of 'perimeter'
 - Introduction of a formula for circumference of a circle without understanding what the formula means; there is no simplification into identifying a circle, and what is meant by its 'circumference', and why this information might be needed in real life.
 - Formula for 'perimeter' taught, but there is no understanding of breaking irregular shapes up. Real life examples are not used, only exercises following the formula

Analysis of the syllabus with notes of rectifications needed to help to overcome deficiencies in understanding and learning

Standard I

- The syllabus jumps from working with numbers 1 to 9 to working with numbers 1 to 99. There is no development of number facts and counting, nor of place value for units and tens. This underpins place value knowledge in other Standards. This is a huge problem.
- Adding with carry numbers: pupils are still developing the facts of numbers and basic processes; they have not been introduced to place value and they are required to add with carrying. This is too advanced.
- Adding and subtracting horizontally with carrying: This is using the 'abstract' too early, and is too advanced at this stage. Place value must be introduced first, with lots of 'concrete' work.
- Number facts and processes must be introduced moving from 'concrete' to 'abstract'. At this level pupils are still very much at the basic 'concrete' level of learning and understanding.

Missing from the syllabus:

- Developing the knowledge of simple ordinal numbers
- Developing 1 to 1 correspondence when counting (e.g. 4 represents four bottles/sticks/buttons etc)
- Developing concepts or ideas in measurement such as longer/shorter
- Developing ideas or concepts in geometry such as sorting objects according to shape
- Developing the concept of time such as 'today', 'yesterday', 'tomorrow' and 'days of the week'
- Ordering numbers between 1 and 20
- Drawing diagrams to show + and - e.g. drawing of four cats and two dogs = 6 animals

Standard II

- Reading & writing numbers to 1,000. This is a big jump from Standard I. Matching number cards with object to 1,000. Who is going to count single objects to 1,000? Where is the teacher to get the objects?
- Adding and subtracting numbers to 1,000 with carrying and borrowing which is a huge jump from Standard I.
- Multiplication is introduced, "Multiply numbers to get a product not exceeding 72." Pupils have not been introduced to the process of multiplication. and yet in the syllabus they are learning the 'x' sign and multiplication charts before they know or understand what the process is or what it achieves.
- The term 'quadrilateral' is used and yet it is too difficult before the idea or concept has been developed. This should be developed by 'concrete' items.

Missing from the syllabus:

- Developing counting skills- skip counting by 2, 5 and 10 (e.g. 2,4,6, 5,10, 15 10,20 30 etc.)
- Place value not mentioned
- Sharing and repeated subtraction (as a valuable introduction to division)
- Counting-on skills (e.g. starting with 6, and counting-on three more)
- Introduction of multiplication using arrays (these are easy diagrams and easy with 'concrete' items)
- Development and use of a 'fact family' linking $25 + 5 = 30$ to $5 + 25 = 30$, $30 - 5 = 25$ and $30 - 25 = 5$
- Working with shapes to create pattern using flips, slides and turns, and terminology left/right, up/down
- Developing thinking strategies
- Further development of the concept of time- days, weeks, months, years

- Introduction to data and graphing using pictograms and human graphs (using pupils to represent information in graph format)
- Odd and even numbers
- Calendars

Standard III

- Reading and writing numbers up to 10,000 is a huge jump from Standard II.
- Adding, subtracting and multiplying are all used with these large numbers. This is too big a jump from Standard II. Place Value has still not been mentioned.
- Division is introduced, “divide whole numbers up to three digits by a number with no more than one digit and no remainder.” Pupils have not been introduced to the idea of division. The process is too advanced and needs an introduction from ‘concrete’ towards ‘abstract’. This is the source of problems in future Standards.
- Problem solving is introduced using data from HIV Aids. This is not suitable for Standard III and hinders the development of the idea of problem solving. This lack of suitable, simple and age-relevant work in this area is the reason for problems in higher Standards. The introduction must be practical, simple and relevant e.g. number of pupils in a class, numbers of boy and girls - There are 54 students in the class. Half the class are girls. How many pupils are girls?
- Fractions are only taught as fractions of objects/shapes. No fractions of numbers are mentioned. This is the cause of problems as seen in the Standard IV national examination in 2012.

Missing from the syllabus:

- Chance/Probability
- Place value not mentioned
- Further development of times tables knowledge e.g. grouping, sharing
- Skip counting by 10, 3, 4, 5, 6
- Development of terminology for geometry e.g. lines, corners, 2D and 3D shapes
- Beginning to develop skills in approximation estimation
- Factors should be introduced as they are linked to times-tables
- HCF and LCM to be introduced using times-tables

Standard IV

- Pupils are working with numbers up to 100, 000. This is a big jump from Standard III and Place Value is only just mentioned. This introduction of Place Value is too late and this is contributing to problems in the development of mathematical knowledge and understanding
- Pupils are asked to match number cards with corresponding piles of objects up to 100,000. This is not practical. Who can supply 100,000 objects, and what is the aim of the exercise?
- Word/narrative problems use HIV data. Pupils have not been introduced to what ‘data’ is and what it means, and so this is not a suitable example. Pupils have not been introduced to reading graphs and interpreting data. There should be more age-appropriate examples e.g. shopping, animals on farms, pupils at the school etc
- Addition of fractions is introduced before other ideas of fractions have been developed and consolidated
- Division- “divide a whole number not exceeding four digits by a divisor of up to two digits.” This is a huge jump from Standard III.

- Area and perimeter are introduced but pupils do not understand the ideas as they are presented in an 'abstract' way. This caused problems in the Standard IV national examination in 2012, and many pupils added area and multiplied perimeter.
- Measurement is introduced using metric units; most schools do not have the equipment to do the practical work so it is introduced and practised in an 'abstract' way. The pupils do not understand what they are doing.
- Standard IV is the first time that daily application of mathematics is mentioned.
- Standard IV is the first time data or graphing is introduced; pupils do not understand this. This caused problems in the national examination in 2012. Most pupils interpreted a pie-graph as a fraction of a shape.
- Invoices are introduced at standard IV. This is not suitable in this Standard, nor indeed at primary level.

Missing from the syllabus:

- Further development of fraction knowledge such as simple equivalent fractions
- Development of knowledge of fact families ($5 \times 7 = 35$, $35 \div 7 = 5$) to solve division problems
- Development of other strategies such as doubles, near doubles, estimation, extended number facts ($4 \times 3 = 12$ so $40 \times 3 = 120$. $9 + 1 = 10$ so $90 + 10 = 100$)
- Geometry – drawing nets of shapes, making 3D shapes, terminology such as 'horizontal', 'vertical', and knowledge of polygons
- Knowledge of inverse operations
- Development of estimation skills
- Development of times tables knowledge
- Development of pattern and order skills. This linked with times table knowledge and special thinking.
- Further development of problem solving strategies e.g. 'guess and check', break the problem down into smaller parts, work backwards.

Standard V

- Factors are introduced but there is no work on what they are. This is linked to times tables and should have been started in standard III.
- Types of numbers - odd, even and prime. Odd and even numbers should be introduced in Standard I or II.
- Square numbers are introduced, "write square numbers not exceeding 100" and "Solve problems involving square numbers." There is no introduction of what a 'square number' is. This is a huge jump, and the 'abstract' is used before the 'concrete' and so no practical understanding is gained.
- Decimals are introduced but there is no reference to Place Value.
- Pictograms are introduced. This is a basic for graphing and should be introduced before bar graphs. These should have been introduced in Standard II
- Algebra is introduced but there is no understanding of what it is or how it is used. This is a huge jump and is the cause of many problems pupils face with algebra.
- Percentage is introduced, "Identify percentage." "Read and write percentages." There is no prior learning about what a percentage is or the link to pie graphs.
- Calendars are introduced. This is a very basic topic which can be introduced in Standard I or II.

Missing from the syllabus:

- Composite numbers
- Times tables, skip counting
- Estimating
- Developing number knowledge short cuts
- Place value with decimals
- What percentage means. What is 'per cent'?

Standard VI

- In the topic of integers the concept of 'opposite' is introduced. Left/right, hot/cold, up/down. This should have been introduced in standard I or II.
- Integers are just introduced and the topic jumps immediately into adding and subtracting.
- HCF and LCM are introduced and this is something that should be done from standard III or IV using times tables
- Introduction of the area of a circle. The way it is done is far too complicated. There are much easier ways which develop understanding. The formula πr^2 is introduced without understanding what it means
- Other formulae are also introduced without the practical understanding e.g. $2\pi r^2$
- Pie graphs are introduced but there is no learning about what they are. The syllabus goes immediately into drawing circle-sectors. Work on identifying Pie Charts is very easy. Of more relevance would be to interpret the data presented in a pie chart. Pie charts are for representing data so the interpretation is what is of importance, e.g. What is the chart telling us? What information do we get from it?
Only if pupils can solve problems using pie charts does it show that the use of pie charts is clearly understood.
The same process is used when introducing line graphs.
- Basic operations with mixed numbers. This is usually done with decimals, which is much easier. Things should not be made hard for pupils when it is not necessary.

Missing from the syllabus:

- Composite numbers
- Times tables, skip counting which should have been introduced in Standard II
- Very little mention of the relationship between decimals, fractions and percentage
- No mention of Place Value with decimals
- Estimating
- Developing number knowledge short cuts
- What percentage means. What is percent?
- Link between coordinates and maps
- Identification of the mathematical information needed to solve a problem or carry out an investigation
- BODMAS (the order of operations: brackets of division, multiplication, addition and subtraction) should be introduced with linear equations

Standard VII

- Basic operations to 1,000,000,000 – doing exercises is mentioned in the syllabus many times. What this means for pupils in the classroom is copying equations from the board. By this stage addition, subtraction, multiplication and division should be integrated into other areas. Mention is made of an abacus, which seems impractical with numbers this large.
- Approximation/estimation; the English used has changed from the introduction on page V of the syllabus. This topic should have been introduced in about Standard III. It is key to developing short cuts, checking calculations and problem solving.
- Algebra - the syllabus says “simple linear equations”. Again there is no real explanation of real life uses for algebra. This is what pupils need in order to have a full understanding of the topic. The solving of many different problems which are set out using only figures is not aiding understanding
- Integers are introduced as a separate topic. This topic should be combined with other topics to make it relevant and understandable.
- BODMAS (the order of operations: brackets of division, multiplication, addition and subtraction) is only just being introduced when linear equations were introduced in Algebra in Standard VI.
- Pythagoras’s Theorem and Distance and Height are introduced as separate topics when they should be placed with other topics. The three topics appear to be placed in the syllabus with no idea of a continuum.
- Volume of solid shapes is introduced separately when it should be linked and incorporated into Geometry.
- Statistics, “..... able to solve graphical problems involving specified data.” In the assessment column of the syllabus is written, “Draw and solve graphical problems.” From the examination papers analysed and what has been observed in classrooms this seems to be an exercise in making problems that require complicated mathematical calculations. What is missing is the real purpose; interpreting graphs, understanding the use of a bar graph, line graph and pie graph.
- Profit and loss- this is a very specialized area and is not really relevant to pupils in Standard VII. It is relevant to economics in secondary school.
- Percentage is introduced in Standard V and yet it is related to pie graphs and the link is not made.

Missing from the syllabus:

- Chance/Probability
- Links between mathematics and the practical uses- e.g. relating maps to the teaching of coordinates, linking fractions with shopping- $\frac{1}{2}$ kg of something.
- Further development of times tables knowledge - grouping, sharing
- Skip counting
- Integration of topics e.g. using measurement when adding, multiplying, subtracting and dividing, and teaching percentage and pie graphs as integrated topics.
- Development of terminology for geometry e.g. lines, corners, vertices, 2D and 3D shapes
- Developing estimation skills; it is mentioned on page V of the introduction to the syllabus and is then not mentioned until Standard VII (and then in the English version the term has been changed to approximation).
- Problem solving process and strategies
- Different teaching methods. Most teaching and learning strategies in the syllabus include:
 - Teacher to demonstrate
 - Teacher to guide
 - Pupils to do exercises
 - Teacher to use question and answer

There is no explanation of what these mean? What has been observed is teachers putting exercises on the board from textbooks, pupils are asked literal questions (questions that require little or no

logical or critical thinking. e.g. What is 4×5 ? instead of, How many ways can you make 20?) so the pupils do not get the opportunity to develop thinking skills. Mathematics is about developing thinking skills so that pupils can apply their knowledge in everyday settings

- Developing problem solving skills

General summary:

Mathematics is taught with much use of numbers and figures alone with rote learning of formulae. Very little is done with concrete items which help to make mathematics real. There is little connection to the real world and the practical day to day use of mathematics. After all, the reason for teaching Mathematics is so we can function in an everyday life.

Some of the learning, and most of the introductions of topics are beyond the actual mental development of the children in that Standard. This results in the pupils, very early on, feeling that mathematics is hard and they cannot do it. This in turn creates a negative mind-set for the rest of their education.

Topics are taught at separate, discrete times, and there is no integration of mathematical ideas, processes or concepts. For example processes with money are taught separately from other processes. When teaching '+', '-', 'x' and '÷' money could be used. Also time or measurement units could be used as the items with which to work.

The syllabus more often than not goes immediately onto 'abstract' concepts before the pupils have an opportunity to develop a sound understanding of the basic mathematical idea or process.

The comments in this paper are set out in general terms. However, the authors of this paper have years of experience in Tanzania's government primary schools and would be willing to contribute to discussions concerning, or development of, the syllabus and resultant textbooks.

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Key for Appendices

N+- Number Strand- addition	NEG + - adding negative numbers
Nx –Number strand – Multiplication	NEG – - subtracting negative numbers
N- Number strand – Subtraction	NEG x - multiplying negative numbers
N÷ Number strand – Division	NEG ÷ - dividing negative numbers
D + - adding decimals	Geo- Geometry
D – - subtracting decimals	Mper- Measurement – Perimeter
D x - multiplying decimals	ML – Measurement – Length
D ÷ - dividing decimals	MA- Measurement – Area
RN- Roman Numerals	Mmas- Measurement –Mass
NF- Number strand- Fractions	P S- Problem Solving
D Sehem- decimals and fractions	% D – percentages and decimals
KDS – Lowest common multiple	Graph
KKS – Highest common multiple	
• Saa – Time	
Mzingo - Perimeter	
Ujazo - Volume	
Nam Tasa - Prime Numbers	
Nam Mraba - Square Numbers	
Eneo - Area	

Appendix 1 - Standard 4 Mathematics

Each column represents a student. Each row represents a question from the exam paper. Each coloured cell with a '1' represents a correct answer to the question. Blank spaces represent an incorrect answer.

Darasa la Nne 2012 Hisabati

Q	Type	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	aa	ab	ac	a
1	N +	1	1		1		1	1	1	1					1		1	1	1		1	1		1	1	1		1	
2	N x												1				1												
3	N -																							1		1			
4	N ÷												1																
5	RN				1				1	1		1	1				1					1		1	1	1	1		
6	RN				1																			1					
7	NF	1			1	1	1			1	1	1	1				1	1	1			1		1	1			1	
8	NF	1			1	1	1			1	1	1	1				1	1	1			1	1	1	1			1	
9	Geo	1	1	1	1		1	1	1	1	1	1	1		1	1			1	1	1	1	1	1	1		1	1	
10	Mper	1			1	1													1	1				1					
11	MA																												
12	Mper						1																						
13	ML	1	1	1	1					1			1				1	1	1			1		1	1			1	
14	ML	1	1	1	1			1	1	1	1	1	1	1	1		1	1	1	1		1		1	1	1	1	1	1
15	ML						1					1		1			1					1						1	
16	Mmas	1	1		1		1			1								1	1					1	1				
17	PS																												
18	ML	1	1				1										1	1	1					1	1			1	
19	PS	1	1				1			1							1	1	1					1	1			1	
20	Time																												1
21	Time	1	1	1	1			1	1	1	1	1	1		1	1	1	1	1		1		1		1	1	1	1	1
22	Graph									1									1						1				
23	Money	1			1	1		1		1			1					1	1	1				1		1		1	
24	Money		1			1		1					1			1			1	1	1	1	1	1	1			1	1
25	PS				1			1		1	1						1	1	1	1	1		1		1		1	1	1

Appendix 2 - Standard 7 Mathematics

Each column represents a student. Each row represents a question from the exam paper. Each coloured cell with a '1' represents a correct answer to the question. Blank spaces represent an incorrect answer.

Darasa la Saba – Mock 2013

Qu	Type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	+	1	1	1	1			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	-	1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
3	x	1	1	1	1			1	1	1	1	1	1				1	1	1	1	1	1	1	1	1	1	1	1	1	1	
4	÷	1	1	1			1				1	1				1			1	1	1	1	1	1	1		1	1	1		
5	D +	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
6	D -	1	1	1	1	1				1					1			1	1				1	1	1		1	1	1	1	
7	D x	1	1	1	1			1	1	1	1	1	1					1				1	1	1	1	1	1	1	1	1	
8	D ÷	1	1	1			1			1	1				1				1	1	1	1	1	1	1	1	1	1	1	1	
9	NEG +			1	1	1													1				1	1	1	1		1	1	1	
10	NEG -	1	1			1	1	1	1	1	1								1				1	1			1	1	1	1	

